Hale School

Mathematics Specialist

Test 3 --- Term 2 2018

Vectors

/ 45

Name: _____

Instructions:

- **Calculators are allowed**
- **External notes are not allowed**
- **Duration of test: 45 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (Otherwise, no marks awarded)**
- **This test contributes to 7% of the year (school) mark**

All arguments must be given using principal values.

1. **[3, 2 = 5 marks]**

(a) Find the acute angle between $\underset{\sim}{r} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ and the line joining the points

P$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and Q$\begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$, giving your answer correct to the nearest degree.

(b) Find an equation of the plane through Q$\begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$ and perpendicular to

$\underset{\sim}{r} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$, in the form $\underset{\sim}{r} \cdot \underset{\sim}{n} = \rho$.

2. [5, 5 = 10 marks]

(a) The shortest distance between the plane $2x - 3y + 4z = 6$ and a parallel plane is 5. Determine the vector equation of the parallel plane.

(b) The plane Π_1 contains the line $\frac{2-x}{3} = -\frac{y}{4} = z + 1$ and is parallel to the vector $3\underset{\sim}{i} - 2\underset{\sim}{j}$. Find the Cartesian equation of Π_1 .

3. [5, 3 = 8 marks]

Given the system of equations

$$x + 2y + mz = -1$$
$$2x + y - z = 3$$
$$mx - 2y + z = 1$$

(a) Determine the value(s) for m for which there is a unique solution.

(b) Determine the value of m for which there are infinite solutions and give geometrical meaning to illustrate this case.

4. [3, 3 = 6 marks]

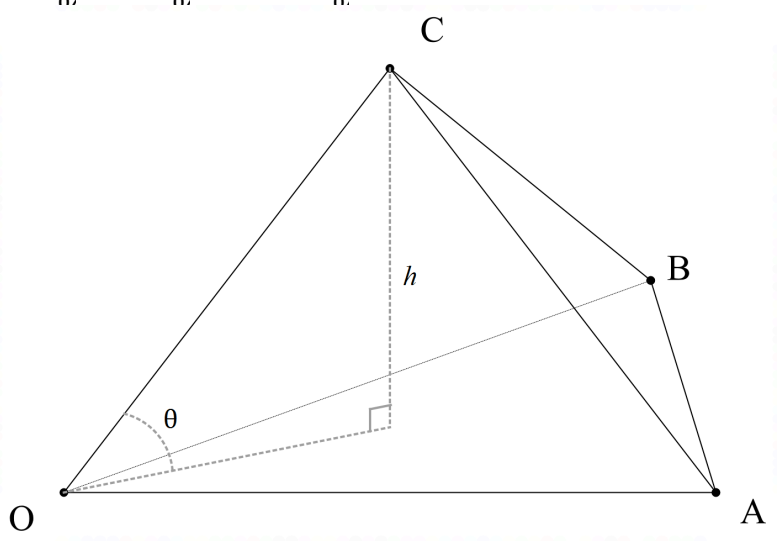
$\left|\underset{\sim}{r}-(1,-1,0)\right|=\sqrt{26}$ is the equation of a sphere.

(a) Find the point(s) where the line through $(4,-2,-2)$ and $(6,2,-4)$ meets the sphere.

(b) A plane touches the sphere at the point $(-2,3,1)$. Determine the Cartesian equation of the plane.

5. [1, 4 = 5 marks]

In the tetrahedron shown, $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$ and $\overrightarrow{OC} = \underset{\sim}{c}$.



(a) Express h in terms of $\underset{\sim}{c}$ and θ .

(b) Given $V = \frac{1}{3} Base \times h$, show that the volume of the tetrahedron can be found by $V = \frac{1}{6}\left|\left(\underset{\sim}{a} \times \underset{\sim}{b}\right) \cdot \underset{\sim}{c}\right|$.

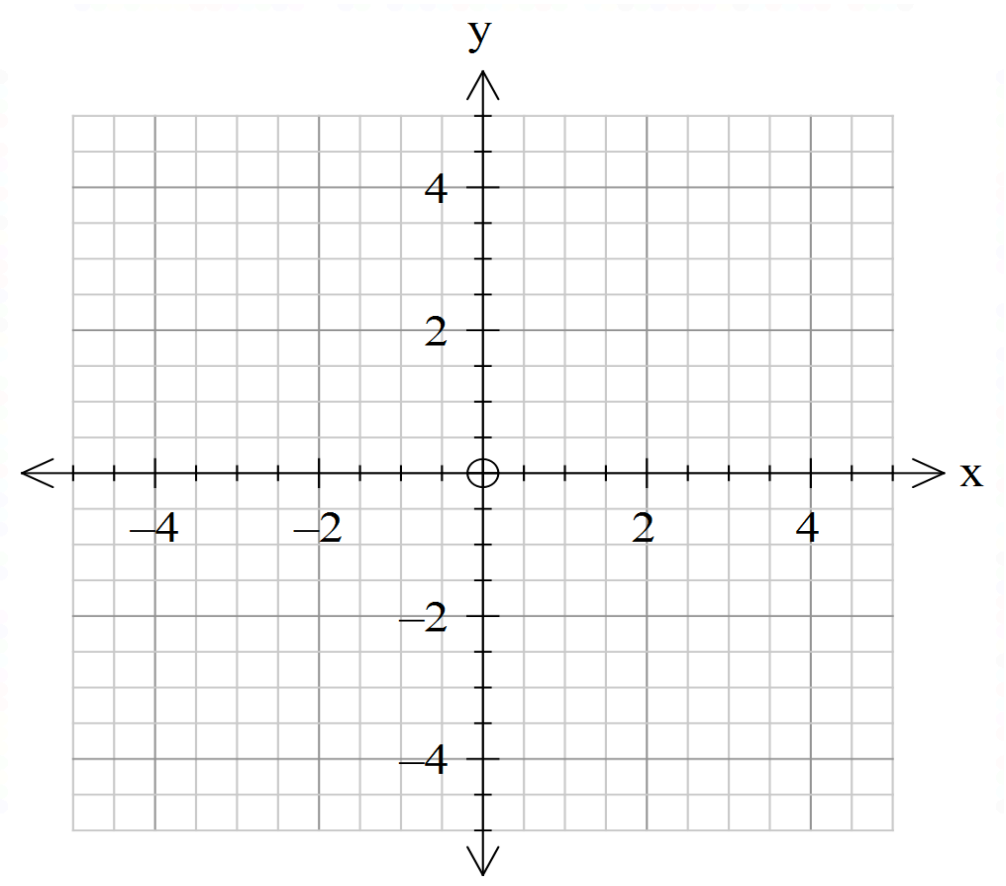
6. **[3, 3 = 6 marks]**

A curve, called the "witch of Maria Agnesi", is defined by the vector equation

$$\underset{\sim}{r} = \frac{2}{\tan(t)}\underset{\sim}{i} + 2\sin^2(t)\underset{\sim}{j}, \quad 0 < t < \frac{\pi}{2}.$$

(a) Determine the Cartesian equation of this curve.

(b) Sketch the curve and indicate the direction of motion.



7. [5 marks]

Particle A starts from $(2,-1,5)$ and has a velocity vector $2\underset{0}{i}-3\underset{0}{j}+6\underset{0}{k}\ ms^{-1}$. Particle B starts 5 seconds later from $(12,-10,6)$ and has a velocity vector $\underset{0}{i}+\underset{0}{j}+2\underset{0}{k}\ ms^{-1}$. Find the time at which the particles are closest together and the minimum distance.

_____End of Test_____